Optional Homework Due April 15th or 16th

Show your work. Answer without work receives no credit.

Quantifiers

1. Circle all options below that are logically equivalent to the proposition "Every rose has thorns and red petals."

(a) Every rose has no thorns or no red petals.
(b) There exists a rose that has no thorns or no red petals.
(c) Every rose has no thorns or every rose has no red petals.
(d) Some rose has no thorns or some rose has no red petals.
(e) Every rose has no thorns and no red petals.
(f) There exists a rose that has no thorns and no red petals.
(g) Every rose has no thorns and every rose has no red petals.
(h) Some rose has no thorns and some rose has no red petals.

Nested quantifiers

Answer Problems 1–10 with true or false and justify your answer with either an example or one short sentence.

2. $\forall x \in \mathbb{N} : [\exists y \in \mathbb{N} : x > y]$
3. $\exists x \in \mathbb{N} : [\forall y \in \mathbb{N} : x > y]$
4. $\exists x \in \mathbb{N} : [\exists y \in \mathbb{N} : x > y]$
5. $\forall x \in \mathbb{Q} : [(x > 0) \Rightarrow (\exists y \in \mathbb{Q} : (y > 0) \wedge (x > y))]$

Translate the following sentences into formal logic. Your answer may only use symbols taken from the following list: $\wedge$, $\vee$, $\oplus$, $\neg$, $\Rightarrow$, $\forall$, $\exists$, : (colon), $[$, $]$, $($, $)$, $\in$, $\mathbb{R}$, $\mathbb{N}$, $\mathbb{Z}$, $<$, $>$, $=$, $\leq$, $\geq$.

6. "For every real number, there is an integer strictly greater than it." (Archimedean Property)
7. "Between any two distinct real numbers x and y, there is another real number strictly between x and y."
8. Negate the sentence: "There is a person who has eaten at every restaurant in Glassboro."

Onto functions

Which function below is onto? **Justify your answer.**

9. $f : \{0, 1, 2, 3\} \to \{0, 1, 2, 3\}$ defined by $f(0) = 3, f(1) = 2, f(2) = 1, f(3) = 0$.
10. $f : \{0, 1, 2, 3\} \to \{0, 1, 2, 3\}$ defined by $f(0) = 1, f(1) = 2, f(2) = 1, f(3) = 2$.
11. Is $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$
12. $g : \mathbb{N} \to \mathbb{N}$, $g(x) = x + 1$
13. $h : \mathbb{Q} \to \mathbb{Q}$, $h(x) = 3x + 1$
14. $j : \mathbb{Z} \to \mathbb{Q}$, $j(x) = x/6$

Hint:
9. Yes
10. No

11. No
12. No
13. Yes
14. No

One-to-one functions

Which function below is one-to-one? **Justify your answer.**

15. The absolute value function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = |x|$
16. $g : \mathbb{Z} \to \mathbb{Z}$, $g(n) = 5n - 8$
17. $h : \mathbb{Z} \to \mathbb{Z}$, $h(n) = n^2$
18. $j : \mathbb{R} - \{0\} \to \mathbb{R} - \{0\}$, $j(x) = 1/x$.

Hint:
15. No
16. Yes
17. No
18. Yes

Bijections (not homework)

Which function below is a bijection? If it is a bijection, find its inverse.

19. $f : \mathbb{Q} \to \mathbb{Q}$, $f(x) = x/7$
20. $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = x^4 + 1$

More practice (not homework)

Let $\mathbb{R} - \{0\}$ be the set of nonzero real numbers.

21. Let $f : \mathbb{R} - \{0\} \to \mathbb{R} - \{0\}$ be the function defined by $f(x) = 1/x$. Is f onto?
22. Let $f : \mathbb{R} - \{0\} \to \mathbb{R}$ be the function defined by $f(x) = (x + 1)/x$. Is f one-to-one?
23. Let $f : \mathbb{R} - \{0\} \to \mathbb{R}$ be the function defined by $f(x) = (x + 1)/x$. Is f onto?